Virtual Power Limiter System
which Guarantees Stability of Control Systems

Katsuya Kanaoka and Mitsunori Uemura
Department of Robotics, Ritsumeikan University
Shiga 525-8577, Japan
Email: kanaoka@se.ritsumei.ac.jp, rr002993@se.ritsumei.ac.jp

Abstract—In this paper, a Virtual Power Limiter System is proposed. This requires no modeling and makes it possible to guarantee the stability of control systems which include unknown characteristics: flexibility in flexible manipulators, compressibility in pneumatic servo systems, human dynamics in man-machine systems, hardware nonlinearity in mechatronic systems, and so on. The details of the proposed virtual power limiter system and experimental verification with a man-machine system are presented.

I. INTRODUCTION

It is a very important problem to guarantee the stability of feedback control systems. Unknown characteristics in the systems badly affect the stability. Some attempts to guarantee the stability and to improve control performances have been done, such as adaptive control or robust control based on dynamics modeling and identification. In the case that the unknown characteristics cannot be estimated at all, however, the only solution seems to be brought by changing the controllers into more conservative ones, usually based on trial and error. It is very difficult to strike a balance between the stability and high performance.

Unlike the above solution, we proposed a Passivity Monitor and Software Limiter [1], which is the framework to stabilize control systems only based on output monitoring and simple online adjustment of controllers. In this paper, we redefine it as a Virtual Power Limiter System (VPLS) in more sophisticated and more general form.

The problem that we should solve is as follows. Assume that a control system as shown in Fig.1 is used by a certain user. We call it an User’s Control System (UCS). An user’s control strategy and an user’s control object in the UCS are defined as in Fig.1. Here, our problem is how to guarantee the stability of the UCS, which can go unstable easily by unknown factors, and how to strike a balance between the stability and high performance without knowing the detail of the USC. The solution is given by adding the VPLS to the UCS.

An extension of the VPLS to man-machine systems is also presented. With the recent expansion of significance of man-machine systems, which have interactions with human, much work has been done. However, it is difficult to guarantee the stability of such man-machine systems, because the systems include unknown dynamic characteristics induced by human and have complex interaction between human, robots and environments. In most of existing work, this problem is dealt with by making the original purpose conservative [7]. In this paper, it is shown that the VPLS enables to guarantee the stability of man-machine systems with unknown factors. Some experimental results using a man-machine system verifies the validity of the VPLS.

II. VIRTUAL POWER LIMITER SYSTEM

In this section, the virtual power limiter system is defined. The VPLS is composed of a virtual power monitor and a virtual power limiter. The VPLS is defined as a system that the virtual power limiter adjusts and stabilizes control systems based on a stability measurer which is evaluated by the virtual power monitor.

A. Virtual Power Monitor

The virtual power monitor is defined as a system that evaluates the stability of the UCS. The virtual power monitor requires a conservative control strategy separately from the user’s control strategy. The detail of the conservative control strategy is defined later. The input from the conservative control strategy \( u_{csv} \in \mathbb{R}^n \) does not necessarily need to be input to the user’s control object. This input \( u_{csv} \) is usually connected virtually to the UCS as shown in Fig.2.

The output \( y \in \mathbb{R}^n \) should be chosen as a conjugate power of \( u_{csv} \).

A monitored value \( P_v \) is defined as the following.

\[
P_v(t) = y^T(t)u_{csv}(t)
\]  

This \( P_v \) means the virtual power transferred from the conservative control strategy to the UCS. The virtual power monitor can evaluate the stability of the UCS by this \( P_v \).

Fig. 1. User’s Control System (UCS)
B. Virtual Power Limiter

The virtual power limiter is defined as a system that regulates the virtual power $P_v$ and stabilizes the UCS by adjusting the UCS based on the $P_v$. Fig. 3 is the conceptual scheme of the virtual power limiter.

III. VPLS DESIGN BASED ON CONCEPT OF PASSIVITY

In this section, the VPLS is designed based on the concept of passivity. Then, it is shown theoretically that the VPLS guarantees the stability of the UCS.

A. Passivity

The concept of passivity [2] is one of the most important characteristics of robot systems. The definition of the passivity is given as follows.

In terms of the input $u \in \mathbb{R}^n$ and the output $y \in \mathbb{R}^n$, the system is called passive if the following inequality is always satisfied,

$$\int_0^t y^T(\tau)u(\tau) d\tau \geq -\gamma_0^2 \quad (\forall t > 0) \quad (2)$$

where $\gamma_0^2$ is a non-negative constant depending on the initial state of the system. Physically, it is the initial energy of the system.

B. Stability Evaluation

In order to evaluate the stability of the UCS by the virtual power monitor, the value $E_v$ calculated from $P_v$ is considered.

$$E_v(t) = \int_0^t P_v(\tau) d\tau = \int_0^t y^T(\tau)u_{csv}(\tau) d\tau \quad (3)$$

This $E_v$ means the virtual energy transferred from the conservative control strategy to the UCS.

The following inequality is introduced for the stability evaluation of the UCS.

$$E_v(t) = \int_0^t y^T(\tau)u_{csv}(\tau) d\tau \geq -E_{v0} \quad (4)$$

where $E_{v0}$ is a constant, which a system designer should define. This inequality is considered as a stretch of the concept of passivity (2). In other words, the UCS which satisfies the equation (4) cannot be distinguished from a passive physical system with the initial energy $E_{v0}$ in terms of the input and output.

Suppose that the following negative feedback is chosen as the conservative control strategy.

$$u_{csv} = K(y_d - y) \quad (5)$$

$K \in \mathbb{R}^{n \times n}$ is a positive diagonal matrix, and $y_d \in \mathbb{R}^n$ is a control command of the conservative control strategy. Then,

$$\int_0^t y_d^T u_{csv} d\tau = \int_0^t (K^{-1}u_{csv} + y)^T u_{csv} d\tau = \int_0^t y^T u_{csv} d\tau + \int_0^t u_{csv} K^{-1} u_{csv} d\tau \quad (6)$$

If (4) is satisfied,

$$\int_0^t y_d^T u_{csv} d\tau \geq -E_{v0} + \int_0^t u_{csv} K^{-1} u_{csv} d\tau \quad (7)$$

the UCS with the VPLS apparently satisfies output-dissipativity. Equation (4) is a sufficient condition for the stability of the UCS with the VPLS, so the stability of the UCS can be evaluated by monitoring $P_v$ and (4).

C. Stability Analysis of UCS with VPLS

Here the stability of the UCS with the VPLS is analyzed using the Popov’s Hyper-Stability Theorem. The UCS is considered as the backward time-variant nonlinear block, and the conservative control strategy is considered as the forward time-invariant linear block. According to the Popov’s Hyper-Stability Theorem, the sufficient conditions for the asymptotic stability of the feedback system of the UCS and the conservative control strategy are:

1) the conservative control strategy is time-invariant and strictly positive real for the input $y$ and the output $u_{csv}$,
2) the UCS satisfies (4) for the input $u_{csv}$ and the output $y$.

These sufficient conditions are restated as the design policy of the VPLS:

1) the conservative control strategy is designed to be time-invariant and strictly positive real for the input $y$ and the output $u_{csv}$,
2) the output of the conservative control strategy $u_{csv}$ is connected to the virtual power monitor and the virtual power limiter.

![Fig. 2. UCS with Virtual Power Monitor System](image)

![Fig. 3. UCS with Virtual Power Limiter System](image)
2) the VPLS can tune some parameters in the UCS as
the UCS satisfy (4) for the input $u_{\text{csv}}$ and the output $y$.

In the design of the VPLS, the conservative control strategy can be designed without considering performances, because its connection is usually virtual. So the latter condition is essential to guarantee the stability by the VPLS.

D. Strategy to Guarantee Stability by VPLS

Physically, the situation that (4) is not satisfied means the UCS generates energy and excess power is flowing out from the UCS. The VPLS requires the assumption that the UCS can be regulated so as to satisfy (4) by the following methods:

1) decreasing the internal generated energy,
2) increasing the internal dissipated energy.

A solution for the first method is to regulate the input from the user’s control strategy, which is generating excess energy. A solution for the second method is to add some tunable energy dissipative elements in the UCS. In many control systems, these solutions can be easily equipped and are far more realistic than considering the all factors which can affect their stability. The virtual power limiter applies some of the above-mentioned methods based on the monitored $P_v$ and regulates the outflow of the virtual power from the UCS.

As shown in III-C, the asymptotic stability of the UCS with the VPLS is guaranteed if the UCS is tuned to satisfy (4), regardless of whether the connection between the UCS and the VPLS is virtual or real.

IV. CASE STUDIES OF VPLS

This section shows two examples of applying the VPLS to the robot control systems that cannot necessarily guarantee the stability due to unknown characteristics.

A. Robot Control System with Unknown Characteristics

The first example of the UCS is a robot control system. The details of the user’s control strategy and the dynamics of the robot hardware as the user’s control object are unknown for the designer of the VPLS. However, the joint displacement $q \in \mathbb{R}^n$ and the joint velocity $\dot{q} \in \mathbb{R}^n$ are measurable by some sensors, and the control input to the joint actuators from the user’s control strategy $u_{\text{usr}} \in \mathbb{R}^n$ is adjustable.

A structure of the VPLS for this robot control system will be Fig.4. The driving force/torque $u$, which is actually applied to the robot hardware, is

$$u = W_{\text{usr}} u_{\text{usr}} + W_{\text{csv}} u_{\text{csv}} + W_v u_v + u_{\text{dis}}$$  \hspace{1cm} (8)$$

$u_{\text{dis}}(t) \in \mathbb{R}^n$ is unknown input such as disturbance. $u_v(t) = -K_v y(t) \in \mathbb{R}^n$ is a negative output feedback connected to the user’s control object. $K_v \in \mathbb{R}^{n \times n}$ is a positive diagonal matrix of the feedback gain. $W_{\text{usr}}(P_v), W_{\text{csv}}(P_v), W_v(P_v) \in \mathbb{R}^{n \times n}$ are positive diagonal matrices of varying weight functions adjusted by the virtual power limiter. The diagonal elements of $W_{\text{usr}}, W_{\text{csv}}$ are greater than or equal to 0 and less than or equal to 1, and $W_{\text{csv}} = I - W_{\text{usr}}$. $I \in \mathbb{R}^{n \times n}$ is the unit matrix.

The following joint PD control is chosen as the conservative control strategy.

$$u_{\text{csv}} = -K_p (q - q_d) - K_d \dot{q}$$  \hspace{1cm} (9)$$

$K_p, K_d \in \mathbb{R}^{n \times n}$ are positive diagonal matrices. $q_d \in \mathbb{R}^n$ is the target of the joint displacement. The target is the same as for the user’s control strategy, as shown in Fig.4.

If you set the output $y = \dot{q}$, a conjugate power pair is made by $y$ and $u_{\text{csv}}$. The stability evaluation (4) is equivalent to the following equation by (8).

$$\int_0^t \dot{q}^T W_{\text{usr}} (u_{\text{usr}} - u_{\text{csv}}) \, dt + \int_0^t \dot{q}^T u_{\text{dis}} \, dt \leq E_{\text{csv}} + \int_0^t \dot{q}^T u_v \, dt + \int_0^t \dot{q}^T W_v K_v \dot{q} \, dt$$  \hspace{1cm} (10)$$

B. Man-Machine System

Next, the system that has some physical interaction between human and robot is considered. This system is called a man-machine system. Generally, the stability of the man-machine system seems difficult to guarantee because the system includes the human dynamics as its unknown characteristics.

A structure of the VPLS for the man-machine system will be Fig.5. For the man-machine system, the human is considered as the conservative control system.

$$u_{\text{csv}} = u_h$$  \hspace{1cm} (11)$$

$u_h$ is the control force/torque which the human applies to the robot hardware physically. So the virtual power $P_v$ becomes the real power here.

The driving force/torque $u$, which is actually applied to the robot hardware, is

$$u = W_{\text{usr}} u_{\text{usr}} + u_h + W_v u_v + u_{\text{dis}}$$  \hspace{1cm} (12)$$

Fig. 4. Unknown Robot Control System with Virtual Power Limiter System
The stability evaluation (4) is equivalent to the following equation by (12).
\[
\int_0^t \dot{q}^T W_{\text{usr}} u_{\text{usr}} \, dt + \int_0^t \dot{q}^T u_{\text{dis}} \, dt \\
\leq E_{v0} + \int_0^t \dot{q}^T u \, dt + \int_0^t \dot{q}^T W_v K_v \dot{q} \, dt \quad (13)
\]

C. Discussion

As mentioned in III-C, (4) is the sufficient condition to guarantee the asymptotic stability of the UCS with the VPLS. If the effect of the user’s control strategy \( \dot{q}^T u_{\text{usr}} \), the effect of the robot hardware \( \dot{q}^T u \), and the effect of the disturbance \( \dot{q}^T u_{\text{dis}} \) are suppressed by the VPLS, the stability is guaranteed.

Equation (10) and (13) are equivalent to (4). Even in the case that the monitored \( P_v \) is going out of (4), you can tune the UCS to satisfy (4) by changing \( W_{\text{usr}} \) smaller and \( W_v \) larger, in both cases of the unknown robot systems and the man-machine systems. This means to realize the automatic gain tuning by VPLS not to make the UCS unstable.

Here the conditions to satisfy (10) or (13) by the VPLS are discussed on the implementation level.

1) The values of \( y \) and \( u_{\text{csv}} \) are reliable.
2) The calculations of \( W_{\text{usr}}(P_v) \), \( W_{\text{csv}}(P_v) \), \( W_v(P_v) \), and (4) in the virtual power monitor/limiter are reliable.
3) In order to make \( W_{\text{usr}} \) small enough, there is an access channel to the robot control input \( u_{\text{usr}} \). The minimum implementation is that the actuators can be turned off by the VPLS.
4) In order to make \( W_v \) large enough, there is an access channel to control the dissipated energy. The minimum implementation is that the robot is equipped with brakes hard enough to satisfy (10) or (13).

Equation (4) is satisfied and the asymptotic stability of the UCS with the VPLS is guaranteed on the above conditions. Note that there are no assumptions of the details of the user’s control strategy, the user’s control object, and the disturbance except their power values \( \dot{q}^T u_{\text{usr}} \), \( \dot{q}^T u \), and \( \dot{q}^T u_{\text{dis}} \) are bounded. Even the passivity of the robot hardware
\[
\int_0^t \dot{q}^T u \, dt \geq -\gamma_0^2 \quad (14)
\]
does not need to be assumed.

V. EXPERIMENT

This section shows an experimental verification of the VPLS using a man-machine system. In this experiment, a power assist system is chosen as the man-machine system. In some existing analyses, which discuss the stability of power assist systems, the power amplification control must be very conservative. This disadvantage results from complex interactions and large dynamics variability of human and environments. It is quite possible that the system goes unstable due to unknown factors such as unmodeled dynamics and unknown characteristics, even though the stability of man-machine systems is absolutely important.

In this experiment, it is shown that the VPLS guarantees the stability of the power assist system in consideration of unknown factors and attains high performance simultaneously only based on output monitoring and simple adjustment of the UCS.
A. Experimental System

The 1-dof experimental system adopted in the experiment is shown in Fig. 6. A handle is mounted at the tip of the link for manipulation. A force sensor is attached on \( l_f = 0.22 \) [m] from the joint of the robot. This force sensor measures the acting force \( f_h \) [N] from human to the robot. The direction of \( f_h \) is perpendicular to the link. \( f_h \) generates torque \( \tau_h = l_f f_h \) [Nm]. The actuator mounted on the joint generates torque \( \tau_r \) [Nm] for assisting purpose. The velocity \( \dot{x} \) [m/s] at the point where \( f_h \) acts on is calculated from joint angular velocity \( \dot{\theta} \) [rad/s] measured by a optical encoder as \( \dot{x} = l_f \dot{\theta} \). The direction of \( \dot{x} \) is also perpendicular to the link. A weight \( m = 2.00 \) [kg] is attached on \( l_c = 0.19 \) [m] from the joint.

B. Method

The control input is given as follows

\[
\begin{align*}
\tau_r &= w_{pa} K_{pa} \ddot{\theta}_c + \dot{m}_l c_g \cos q \quad (15) \\
\ddot{\theta}_c(t) &= \ddot{\theta}_c(0 - T) \quad (16) \\
\ddot{\theta}_c(s) &= \frac{\omega_c}{s + \omega_c} \ddot{\theta}_c(s) \quad (17)
\end{align*}
\]

where \( w_{pa} \) [-] is a weighting variable adjusted by the VPLS, \( K_{pa} \) [-] is an amplification gain, \( m_l \) [kg] is a link mass, \( g \) [m/s²] is the gravity acceleration, \( T \) [s] is a delay time and \( \omega_c = 8.00\pi \) [rad/s] is a cut off frequency of low pass filter. The reason why the time delay is considered is to simulate actuators such as pneumatic actuators.

The monitored power discussed in IV-B is calculated as follows.

\[
P_v = \dot{x} f_h \quad (18)
\]

In this experiment, the stability is evaluated by \( E_v \) described as follows.

\[
E_v = \int_0^t P_v \, dt \quad (19)
\]

where \( w_{pa} \) is adjusted in the following way by the virtual power limiter.

\[
w_{pa} = \begin{cases} 
1 & \left( E_v > E_u \right) \\
\frac{1}{2} \left( 1 - \cos \left( \frac{E_v - E_u}{E_t - E_u} \right) \pi \right) & \left( E_u \geq E_v \geq E_t \right) \\
0 & \left( E_v < E_t \right)
\end{cases} \quad (20)
\]

where \( E_u = 0.00[J], E_t = 3.93[J] \) \((E_u \geq E_t \geq -E_{v,0})\) are constants chosen appropriately. These equations (20) can be described as Fig. 7. The simple adjustment of amplification gain by the virtual power limiter enables to prevent excess energy transfer from the robot to human and to satisfy the equation (4).

A criterion \( F_{int} \) is introduced for evaluating the amount of force required for human in whole motion.

\[
F_{int} = \int_0^t |f_h| \, dt \quad (21)
\]

Small \( F_{int} \) means the human manipulates the system easily, so the power assist effect can be evaluated by the \( F_{int} \).

Because experiments are carried out by human, completely equivalent results can’t be obtained. So the \( F_{int} \) includes small fluctuation in each experiment. However similar tendency of the \( F_{int} \) in the same experiments is obtained, so this fluctuation is not essential. Six types of experiments are carried out with each setting as shown in Table I. The amplification gain in the case 1-b is chosen as the best setting in order to make \( F_{int} \) smallest without the VPLS.

Each experiment is done for 30 [s] except for the case 2-a. The case 2-a is stopped at 10 [s] for avoiding danger caused by instability. The system is manipulated up and down at 1 [Hz] as far as possible for the first 10 [s] and the last 10 [s], at 2 [Hz] in the middle 10 [s].

C. Result

Fig.8(a) shows the result of the joint velocity trajectory in the case 1-a and the similar results are obtained in the case 1-b, 2-b, 3-a and 3-b. Fig.8(b) shows the result of the joint velocity in the case 2-a.

In the case 1-a, energy is transferred from human to the robot as shown in Fig.8(c) and increase rate of \( F_{int} \) is faster than other cases in whole as shown in Fig.8(i). In the case 2-a, high amplification gain causes oscillation. This brings about energy transfer from robot to human and significant increase of \( F_{int} \) as shown in Fig.8(j). On the contrary, in the case 2-b, oscillation and significant increase of \( F_{int} \) are avoided as shown in Fig.8(j). This effects are brought about by preventing energy transfer based on monitoring energy transfer and reducing the amplification gain. This process means that the VPLS detects the instability and stabilize the system by adjustment of the controller. In the middle 10 [s] in the case 2-b, energy transfer from the robot to human is caused by fast motion as shown in Fig.8(d). The VPLS also prevents this energy transfer. These energy transfer prevention enables to satisfy the equation (4) as shown in Fig.8(g). The last 10 [s] in the case 2-b, the amplification gain recovers as a result of detection of energy transfer from human to the robot.

<table>
<thead>
<tr>
<th>case</th>
<th>( K_{pa} )</th>
<th>VPLS</th>
<th>( T )</th>
<th>Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-a</td>
<td>0.0</td>
<td>without</td>
<td>0.00</td>
<td>No assist</td>
</tr>
<tr>
<td>1-b</td>
<td>1.6</td>
<td>without</td>
<td>0.00</td>
<td>Best setting without VPLS</td>
</tr>
<tr>
<td>2-a</td>
<td>6.0</td>
<td>without</td>
<td>0.04</td>
<td>Excessive amplification with VPLS</td>
</tr>
<tr>
<td>2-b</td>
<td>6.0</td>
<td>with</td>
<td>0.04</td>
<td>Including time delay with VPLS</td>
</tr>
<tr>
<td>3-a</td>
<td>2.0</td>
<td>without</td>
<td>0.04</td>
<td>Including time delay with VPLS</td>
</tr>
<tr>
<td>3-b</td>
<td>2.0</td>
<td>with</td>
<td>0.04</td>
<td>Including time delay with VPLS</td>
</tr>
</tbody>
</table>
In this way, the VPLS adjusts the controller depending on the situation. The result of $F_{int}$ in the case 2-b is finally a little larger than that in case 1-b. This problem is caused by excessive reducing $w_{pa}$ as shown in Fig. 8(g). It is expected that this problem will be solved by tolerating moderate energy transfer from the robot to human, because increase rate of $F_{int}$ in Fig. 8(i) is slower than that in Fig. 8(j) even though the energy is moderately transferred from the robot to human. However, more careful analyses are necessary for this solution, so this will be our future task.

During the first 10 [s] in the case 3-a,b, owing to slow motion, energy tend to be transferred from human to the robot as shown in Fig. 8(e) and the system is controlled stably. Then the increase rate of $F_{int}$ is slower owing to effectiveness of power assist control. Next 10 [s] in the case 3-a, the system goes unstable due to fast motion and energy is rapidly transferred from the robot to human. This energy transfer denotes that $f_h$ is opposite in sign to $\ddot{x}$ and the robot pushes human actively. This means that the system moves contrary to the human will and this causes significant increase of $F_{int}$ as shown in Fig. 8(k).

On the contrary, in the case 3-b, significant increase of $F_{int}$ is avoided owing to preventing this energy transfer as shown in Fig. 8(h). This prevention results from adjusting the amplification gain $w_{pa}K_{pa}$ based on $E_v$. This means that the VPLS can stabilize the system even if the cause of instability is time delay. In the last 10[s] in the case 3-b, the amplification gain recovers owing to detection of energy transfer from human to robot. This gain tuning process in the case 3-b brings about smaller $F_{int}$ than that in the case 3-a.

**D. Discussion**

It is said that power assist systems are difficult to find out amplification gains which can stabilize the control systems and to deal with condition variability due to problems such as force feedback, large dynamics variability, time delay, complex actuator characteristics and so on. This experimental results show the VPLS can guarantee the stability to the power assist system anytime only based on output monitoring and simple adjustment of the controller.

In addition, it is expected that the advantages of the VPLS are applicable to other man-machine systems that
seem to be more important in the near future.

VI. DIFFERENCES FROM SIMILAR STRATEGIES

The similar systems, which guarantee the stability by monitoring the energy balance of a specific control system have been already proposed as the energy balance monitor [3] and the time domain passivity observer/passivity controller [4], [5], [6]. Fig. 9 shows the concept of these similar strategies. These guarantee the stability of the UCS by monitoring the energy balance and regulating the user’s control strategies apparently passive, based on the assumption of the passivity of the user’s control object. Namely, the stability cannot be guaranteed if the user’s control object (with the disturbance) is not passive. Although robot hardwares are originally passive, actual robot systems are not necessarily passive due to low-level controller or disturbance.

Fig. 10 shows the concept of the VPLS. Unlike the similar strategies, the VPLS monitors not only a part but also the whole of the UCS including the control object and environment. This is realized by adding the conservative control strategy and monitoring the virtual power. The VPLS guarantees stability, even if actual robot systems are not passive due to low-level controller, the disturbance, or other factors.

There is another advantage of using two or more control strategies. The user’s control strategy can be aggressive only for the control performance. That is, if the control performance is improved, the nonpassive control strategy is also permitted. Since the conservative control strategy does not participate in actual control if its connection is virtual, it is not necessary to consider the control performance in the design of the conservative control strategy.

On the other hand, the other similar systems need to guarantee the passivity of the user’s control strategy itself. Therefore only the conservative performance can be realized.

VII. CONCLUSION

In this paper, we redefined the virtual power limiter system and described the theoretical analysis. The VPLS is a new framework to guarantee the stability control systems that include unknown characteristic. Experimental results show the validity that the VPLS enables to strike a balance between stability and control performance. It was already shown on the flexible manipulator [1] that the VPLS can stabilize the real robotic systems.

REFERENCES